FC PR 2.8

FC 2-8 a) A GUN IS FIRED STRAIGHT UP. IF $F_D = kmv^2$ SHOW

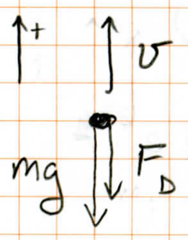
$$v^2 = Ae^{-2ky} - \frac{g}{k}, \quad A = \frac{g + kv_0^2}{k} \quad \text{UPWARD}$$

$$v^2 = \frac{g}{k} - Be^{2ky}, \quad B = \left(\frac{g - kv_0^2}{k}\right)e^{-2ky_0}$$

b) SHOW THAT WHEN THE BULLET HITS THE GROUND

$$v_{HIT} = \frac{v_0 v_t}{(v_0^2 + v_t^2)^{1/2}}, \quad v_t = \left(\frac{g}{k}\right)^{1/2}$$

UPWARD



$$\Sigma F = m\frac{dv}{dt} = mv\frac{dv}{dx}$$

$$-\cancel{m}g - k\cancel{m}v^2 = \cancel{m}v\frac{dv}{dy}$$

$$\int_{v_0}^{v} \frac{-v\,dv}{g + kv^2} = \int_0^y dy$$

$$\frac{1}{k}\ln(g + kv^2\Big|_{v_0}^{v} = -y$$

$$\frac{1}{k}\ln\left(\frac{g + kv^2}{g + kv_0^2}\right) = -y$$

$$\Rightarrow \quad \frac{g + kv^2}{g + kv_0^2} = e^{-ky}$$

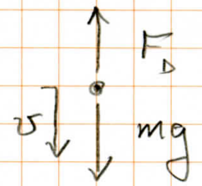
SOLVING FOR v

$$v^2 = \frac{1}{k}\left[(g + kv_0^2)e^{-ky}\right] - \frac{g}{k}$$

$$v^2 = \frac{g + kv_0^2}{k}e^{-ky} - \frac{g}{k} \qquad A = \frac{g + kv_0^2}{k}$$

$$\boxed{v^2 = Ae^{-ky} - \frac{g}{k}} \qquad A = \frac{g + kv_0^2}{k}$$

DOWNWARD



$$\Sigma F = ma$$

$$k\cancel{m}v^2 - \cancel{m}g = \cancel{m}v\frac{dv}{dy}$$

$$\int_{v_0}^{v} \frac{-v\,dv}{g - kv^2} = \int_{y_0}^{y} dy$$

$$\frac{1}{2k}\ln(g - kv^2)\Big|_{v_0}^{v} = y - y_0$$

$$\ln\left(\frac{g - kv^2}{g - kv_0^2}\right) = 2ky - 2ky_0$$

$$g - kv^2 = (g - kv_0^2)(e^{2ky - 2ky_0})$$

$$v^2 = \frac{g}{k} - \underbrace{\left(\frac{g - kv_0^2}{k}e^{-2ky_0}\right)}_{B}e^{2ky}$$

$$B = \frac{g - kv_0^2}{k}e^{-2ky_0}$$

$$\boxed{v^2 = \frac{g}{k} - Be^{2ky}}$$

FIND v WHEN IT HITS THE GROUND

- FIND v_t FROM DOWNWARD MOTION, $v_{0,DOWN} = 0$, $y_0 \to \infty$

$$v_t^2 = \frac{g}{k} - B\overset{\to 0 \text{ AS } y \to \infty}{\cancel{e^{2ky}}} \Rightarrow \boxed{v_t = \sqrt{\frac{g}{k}}}$$

- FIND y_0 IN DOWNWARD MOTION FROM y_{TOP} IN UPWARD

$$v_{TOP}^2 = \left(\frac{g + kv_0^2}{k}\right)e^{-2ky_{TOP}} - \frac{g}{k} = 0$$

$$e^{-2ky_{TOP}} = \frac{g}{k}\left(\frac{\cancel{k}}{g + kv_0^2}\right) = \frac{g}{g + kv_0^2}$$

$$y_{TOP} = -\frac{1}{2k}\ln\left(\frac{g}{g + kv_0^2}\right)$$

2) Continued (FC 2-8)

In v^2_{DOWN} set $y_0 = y_{TOP}$, $v_0 = v_{TOP} = 0$, $y = y_{FINAL} = 0$

$$v^2_{DOWN} = \frac{g}{k} - \left(\frac{g - \cancel{kv_0^2}^{\,0}}{k} e^{-2k\left[-\frac{1}{2k}\ln\left(\frac{g}{g+kv_0^2}\right)\right]}\right) e^{2k\cancel{y}^{\,0 \text{ AT GROUND}}}$$

$$v^2_{HIT} = \frac{g}{k} - \frac{g}{k}\left(\frac{g}{g+kv_0^2}\right)$$

$$= \frac{g}{k}\left(1 - \frac{g}{g+kv_0^2}\right) = \frac{g}{k}\left(\frac{\cancel{g} + kv_0^2 - \cancel{g}}{g+kv_0^2}\right)$$

$$= \frac{g}{k}\left(\frac{kv_0^2}{g+kv_0^2}\right)$$

$$= \frac{g}{k}\left(\frac{v_0^2}{\frac{g}{k} + v_0^2}\right)$$

Noting $v_t^2 = \frac{g}{k}$ gives

$$\boxed{v^2_{HIT} = \frac{v_t^2 v_0^2}{v_t^2 + v_0^2}}$$

Ta Da!